PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Contributions to the Discussion on Astronomy in Ancient Literate Societies

A. Digby and F. R. Stephenson

Phil. Trans. R. Soc. Lond. A 1974 **276**, 117-121
doi: 10.1098/rsta.1974.0013

Contributions to the discussion on astronomy in ancient literate societies

A. Digby (*The Paddocks, Eastcombe, Stroud, Gloucestershire*) I think the most striking difference between Old World and New World calendrical systems which has emerged from the papers today is the use in America of a 260-day period combining the 20 day names with the numerals 1 to 13. There is a possibility that the number thirteen may have been determined by the characteristics and shortcomings of a peculiar sundial which could be used to determine annual as well as diurnal time by showing the declination as well as the hour angle of the sun.

The evidence for the existence of this instrument lies in examples of the year glyph which can be shown to be a drawing of two trapezes set at right angles on a ring. One lying north to south would cast a shadow which would move from west to east across the base of the instrument between the hours of about 7 a.m. and 5 p.m., while the other, a taller trapeze would cast a shadow that travels across the instrument in a direction from south to north and back reflecting the declination of the Sun. There is some evidence to show that the instrument was tilted with the base parallel to the axis of the Earth like a 'polar sundial' (paper read to a symposium on recent Mesoamerican research, Cambridge 1972). Under these conditions, the shadow would make four traverses of equal length in the course of the year: from the centre of the instrument to the northern extremity (autumnal equinox to winter solstice); north extremity to centre (winter solstice to vernal equinox); centre to south extremity (vernal equinox to summer solstice); south extremity to centre (summer solstice to autumnal equinox), each of $91\frac{1}{4}$ days duration.

There must have been some scale to the instrument, but the only drawing known is the drawing of the game of Patolli, shown in Codex Maggliabechiano. The board is cruciform, the arms of the cross have seven divisions, the two outermost of which are marked with a diagonal cross. The movement of beans, representing counters in the game is said to represent the movement of heavenly bodies, and Caso says the Patolli board was used to represent the 52-year cycle common in Mesoamerica.

If we can show that the Patolli board was derived from a scale from the instrument, we have a possible explanation of the origin of the thirteen-day period in the Mesoamerican calendar.

Experiments to determine the accuracy with which the difference in the position of the shadow at noon on two successive days could be detected gave surprisingly accurate results. It was impossible to determine the exact point at which the edge of the penumbra passed over a line on white paper, but once over the line, the penumbra appeared to be clearly separated from the umbra. The contrast between the black line on the paper and the light shadow of the penumbra seemed to emphasize the difference between the two parts of the shadow. It was then easy to determine the point at which the shadow touched the black line. There is no time to describe these experiments but I should be pleased to give fuller details to anybody interested. The results of 94 observations with different observers under conditions of English sunshine gave a mean of 0.29 mm with a standard deviation of 0.09 mm when the cast of the shadow was 20 cm, representing a movement of about 5′. Under conditions of Mexican sunlight the results might be better but pre-Columbian equipment poorer. If we take a less optimistic measure and assume an accuracy to about 10′, the daily variation between the positions of the shadow would not be detectable for about 25 days before and after each solstice. This would explain the differentiation made in the divisions on the Patolli board, and give five divisions signifying fast movement and two signifying slow movement.

The hypothesis offers a plausible explanation of the thirteen day period but further examples of the Patolli board need to be studied, and above all a long series of experiments to determine the accuracy which can be obtained with the trapeze instrument under Mexican conditions of sunlight.

F. R. Stephenson (*Department of Geophysics and Planetary Physics, School of Physics, University of Newcastle upon Tyne*)

Late Babylonian observations of 'lunar sixes'

British Museum tablet 34075 (Sp. 171) is one of the few published late Babylonian texts devoted exclusively to observations of Lunar Sixes. The date of the tablet is clearly specified, and more than 100 complete observations covering a period of 4 years are recorded. Analysis of this tablet provides valuable information on the accuracy of timekeeping in Babylon, and leads to speculation on the design and mode of operation of Babylonian clepsydras.

(1). *Provenance and date*

Photographs of the tablet, together with a full transliteration, translation and commentary, will be published elsewhere. An excellent drawing by T. G. Pinches is shown by Sachs (1955, p. 225). Sachs (1955, p. vi) asserts that the collection to which the tablet belongs originated in Babylon. The very first entry allows us to date the tablet precisely. We read:

[*m*]*u* 1 *kám m pi* [ ],

i.e. 1st year of Philip. Alexander died on 29 Airu in the 14th year of his reign (10 June 323 B.C.– cf. Bickerman (1968)). The astronomical observations preserved to us begin on the following day, namely 1 Simanu.

At least five successive years were originally covered by the text, but the actual period may well have been much longer. Complete observations have only survived for portions of the years 323/322, 322/321, 321/320 and 320/319 B.C.

(2). *Summary of contents of text*

In each month the following six time intervals are recorded:

(i) sunset to moonset on the 1st day of the month (designated *na*);
(ii) moonrise to sunset on the last evening that the (almost full) Moon rose before sunset (*me*);
(iii) moonset to sunrise on the last morning that the Moon set before sunrise (*šú*);
(iv) sunset to moonrise on the first evening that the Moon rose after sunset (*ge*$_6$);
(v) sunrise to moonset on the first morning that the Moon sets after sunrise (*na*);
(vi) moonrise to sunrise on the last morning that the waning crescent was visible (*kúr*).

Each entry starts with the day of the month, the first day entry also naming the month. Intervals of time are quoted in *uš*, the time required for the heavens to turn through 1° (i.e. 4 min). Of the various remarks which may conclude a particular entry, most relate to weather conditions. The common expressions *a* (mist) and *dir* (cloud) are frequently accompanied by *nu pap* (not watched for). In such cases the required interval has been predicted by a procedure which is not yet fully understood, but which made use of the corresponding observational data from 18 years previously (A. J. Sachs, personal communication).

The ideogram *muš* (accurately measured) also commonly follows *a* or *dir*, and indicates a successful observation despite unfavourable weather.

I have confined my attention to those records which either make no mention of mist or cloud, or else are qualified by *muš* – i.e. exclusively observational material.

(3). *Computational procedure*

The majority of the measurements recorded on the tablet are quoted to the nearest *uš*. At first and last visibility (*na* and *kúr*), intervals are almost exclusively rounded to the nearest integer. Evidently it was not possible to achieve greater accuracy, for the narrow lunar crescent is difficult to observe. However, around full moon (*me*, *šú*, ge_6, *na*) the observers seem to have aimed at a significantly higher precision. More than one quarter of these measurements are quoted to the nearest $\frac{1}{2}$ *uš* (i.e. 30/60), with a few scattered results expressed (probably without justification) to the nearest 1/3 or even 1/6 *uš*.

For purposes of comparison, a computer program was designed which calculated the interval between the rising or setting of the Sun and Moon (as required) at Babylon, the geographical coordinates of which were taken as 32° 33′ N, 44° 25′ E. This program required only the input of a selected Julian calendar date and the approximate local time of the phenomenon (6 or 18 h), together with a code number (1 or 2) representing the phase of the Moon (i.e. new or full). Computed intervals were expressed in *uš* to enable direct comparison with the recorded intervals.

Allowance was made for lunar parallax, refraction and dip. In making the correction for dip, it was assumed that the observer was some 15–20 m above the ground (roughly the height of the walls of Babylon – Ravn 1942) but this effect is fairly small for any reasonable height. The moment of rising or setting of the Sun was taken to be the instant when the upper limb (apparently) reached the horizon. There seems no reason to doubt that the Babylonian astronomers, like their present day counterparts, adopted this definition. However, the possibility of an alternative definition will be discussed in §4 below.

In computing the various time intervals, the values of the rotational acceleration of the Earth ($\dot{\omega}/\omega$) and orbital acceleration of the Moon ($\dot{n}$) employed were those derived by the author from an analysis of ancient observations (Stephenson 1972). The results of this investigation were

$$10^9\,\dot{\omega}/\omega = -25.6 \pm 1.2\ (\text{s.e.})\ \text{century}^{-1}$$
$$\dot{n} = -34.2 \pm 1.9\ (\text{s.e.})\ ''/\text{century}^2$$

These figures correspond to a mean rate of the lengthening of the day of 2.2 ms per century and a rate of recession of the Moon from the Earth of about 5 cm per year. The mean epoch of the ancient observations analysed was 100 B.C., not too different from the epoch of the observations recorded on the tablet under discussion. The above results for $\dot{\omega}/\omega$ and $\dot{n}$ agree well with those obtained by de Sitter (1927) from similar material. If the higher values $10^9\,\dot{\omega}/\omega = -27.7 \pm 3.4$, $\dot{n} = -41.6 \pm 4.3$ derived by Newton (1970) from ancient observations of mean epoch 200 B.C. are assumed, my computed local apparent (i.e. sundial) times should be advanced by about 0.4 *uš*. The implications of such an amendment will be considered in §4.

Prior to the operation of the computer program, the Babylonian dates were first corrected to the Julian calendar. The tables of Parker & Dubberstein (1956) were used to obtain the approximate date (never more than a day early or late) and the correct date was established by a trial and error process. In general, errors of observation were found to be small (typically about 1 *uš*) compared with the mean interval between two successive risings or settings of the Moon (about 13 *uš*) so that the true Julian dates were readily obtained, even when the relevant portion of the tablet bearing the date of observation was missing

(4). *Results of analysis*

As expected, the results of analysis of the data recorded on the tablet show that the new Moon observations were much less precise than those at full Moon (by a factor of about 3). It can have been rare indeed that the Babylonians were able to observe the actual rising or setting of the thin crescent, for atmospheric absorption is severe at a low altitude and the crescent is silhouetted against a bright sky. Most measurements probably made use of interpolation. In what follows I have concentrated exclusively on the full Moon data. This is of a high standard.

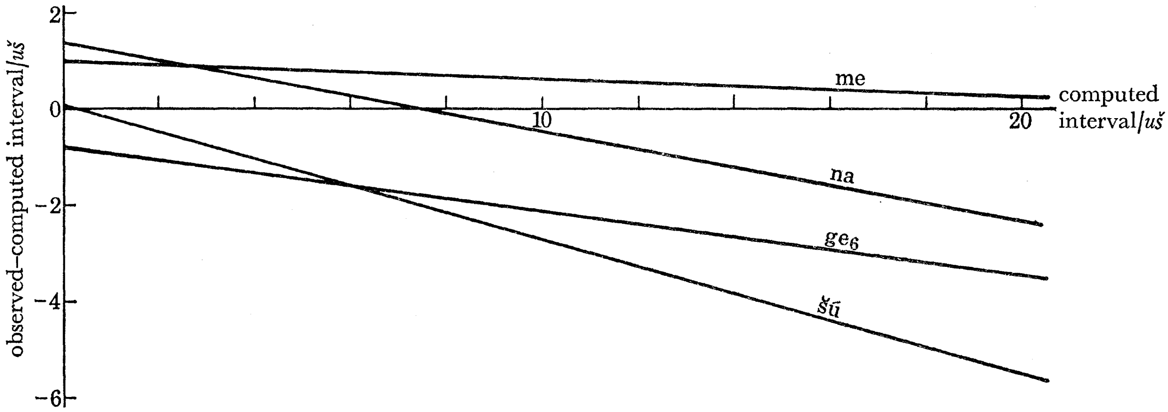

FIGURE 1. Detailed analysis of full Moon observations.

A total of 77 useful full Moon observations are recorded on the tablet. These observations are represented diagrammatically in figure 1, which is a graph of the observed–computed interval (in *uš*) as a function of the computed interval for each of the four categories *me*, *šú*, ge_6, *na*. Each line represents the best least squares fit to the appropriate set of data, after rejecting a few obviously faulty measurements (six in all). Individual points are not shown in order to avoid confusion. Rather surprisingly the four sets of data are quite distinct, both in intercept and gradient. The intercepts and gradients of the lines are as follows (assuming equations of form $y = a + bx$).

$$
\begin{aligned}
&me: & a &= +0.9 \pm 0.2, & b &= -0.04 \pm 0.03,\\
&šú: & a &= 0.0 \pm 0.4, & b &= -0.27 \pm 0.04,\\
&ge_6: & a &= -0.8 \pm 0.4, & b &= -0.13 \pm 0.05,\\
&na: & a &= +1.4 \pm 0.3, & b &= -0.18 \pm 0.03.
\end{aligned}
$$

Let us consider the intercepts and gradients separately. Ideally, we should expect all four intercepts to be zero. Finite intercepts could arise from one of three causes: an incorrect definition of the rising or setting of the Sun and Moon, errors in the adopted values of $\dot{\omega}/\omega$ and $\dot{n}$, or the design of the clepsydra. In the latitude of Babylon, the Sun and Moon rise or set in about 2.5 min. Hence if the Babylonians defined the rising or setting of the Sun and Moon as the moment when the lower limb touched the horizon, my computed intervals *šú* and ge_6 should be increased by about 1.3 *uš* while the intervals *me* and *na* should be diminished by the same amount. The four intercepts in the diagram would thus become:

$$+2.2\ (me),\ -1.3\ (\check{s}\acute{u}),\ -2.1\ (ge_6)\ \text{and}\ +2.7\ (na).$$

Hence in order to minimize the intercepts we must accept the definition of rising or setting as the moment when the lower limb touches the horizon. This, of course, represents the very first or last sighting of the luminary.

The errors introduced by the uncertainties in the adopted values of $\dot{\omega}/\omega$ and $\dot{n}$ would seem to be comparatively small. The effect of alteration to these values is either to advance or delay both the apparent local time of moonrise and moonset. Thus for any random values of $\dot{\omega}/\omega$ and $\dot{n}$ we should expect the intercepts *me* and *šú* to be equal but opposite to the intercepts ge_6 and *na*. This is, of course, not apparent in the results of analysis. It would seem that the intercepts arise mainly from the design of the clepsydra.† This point will be considered below.

The gradients purely represent drifts in the clepsydra (they are in no way dependent on the choice of values of $\dot{\omega}/\omega$ and $\dot{n}$), but it is difficult to explain why the differences should be so marked and so systematic. These variations cannot be explained if an outflow type of clepsydra (whether a cylinder or a frustrum of a cone) is assumed.

The author is of the opinion that the existence of different intercepts, the remarkably small errors in individual measurements and the differences in gradient can most satisfactorily be explained by the assumption of four collecting vessels graduated in *uš*, each reserved for measuring a specific interval (possibly inscribed with the name of the interval). The intercepts can be then explained merely by an error in the marking of the first division (equivalent to a zero error). Assuming the graduations on all four vessels were equal, the varying gradients would be (wholly or partially, depending on whether or not a constant head device was used) the result of slightly differing diameters (the volume, of course, varying as the square of the diameter). From Proclus (*Hypotyposis Astronomicarum Positionum*, IV, 71–81, 87–99) we learn that in the time of Hero of Alexandria (perhaps 1st century B.C.) the apparent diameter of the Sun was measured by starting to collect water from a water clock at the moment of sunrise and continuing until the Sun had completely risen above the horizon. This quantity of water was then compared with the amount collected in a whole day. It is presumed that the Babylonians employed a similar mode of operation, every month waiting until all four samples had been collected before measuring the depth of liquid in each (hence the need for four vessels). We may suppose that, for some unknown reason, the glaring inconsistencies between the rates of filling of the collecting vessels passed unnoticed.

Analysis of other late Babylonian tablets recording lunar sixes may shed valuable light on this problem.

References (Stephenson)

Bickerman, E. J. 1968 *Chronology of the ancient World*, p. 38. London: Thames and Hudson.
Newton, R. R. 1970 *Ancient astronomical observations and the accelerations of the Earth and Moon*. Baltimore: John Hopkins Press.
Parker, R. A. & Dubberstein, W. H. 1956 *Babylonian chronology 626 B.C. to A.D. 75*, Brown University Press.
Ravn, O. E. 1942 *Herodotus' description of Babylon*, trans. M. Torborg-Jensen, p. 20. Copenhagen: Arnold Busck.
Sachs, A. J. 1955 *Late Babylonian astronomical and related texts*. Providence, R.I.: Brown University Press.
Sitter, W. de 1927 *Bull. astron. Inst. Neth.* **4**, 21–38.
Stephenson, F. R. 1972 Some geophysical, astrophysical and chronological deductions from early astronomical records. Ph.D. thesis, University of Newcastle upon Tyne.

† Some kind of clepsydra was definitely used, for during the intervals ge_6 and *šú*, both the Sun and Moon are below the horizon, thus precluding other types of measuring time.